

10.9

POLAR EQUATION OF CONIC
WITH FOCUS @ POLE &
DIRECTRIX p UNITS FROM FOCUS

	DIRECTRIX	VERTICAL	HORIZONTAL
+	RIGHT/ ABOVE	$r = \frac{ep}{1 + e \cos \theta}$	$r = \frac{ep}{1 + e \sin \theta}$
-	LEFT/ BELOW	$r = \frac{ep}{1 - e \cos \theta}$	$r = \frac{ep}{1 - e \sin \theta}$

$$r = \frac{ep}{1 \pm e \begin{matrix} \cos \theta \\ \sin \theta \end{matrix}}$$

+ x, + y
DIR IN POSITIVE TERRITORY $\rightarrow +$
NEGATIVE $\leftarrow -$
- x, - y

cos $\longleftrightarrow$ VERTICAL ($x = p$)
sin $\longleftrightarrow$ HORIZONTAL ($y = p$)

DIR IS

FIND POLAR EQ'N OF CONIC WITH FOCUS @ POLE + EITHER A VERTICAL

①

ECCENTRICITY = $\frac{3}{2} \rightarrow$ HYPERBOLA $e > 1$

OR HORIZONTAL
DIRECTRIX

DIRECTRIX $y = -4 \rightarrow p = 4$

$$r = \frac{ep}{1 - e \sin \theta} = \frac{\frac{3}{2} \cdot 4}{1 - \frac{3}{2} \sin \theta} = \frac{12}{2 - 3 \sin \theta}$$

$\uparrow$ $\uparrow$
 -4 $y =$

$$r = \frac{12}{2 - 3 \sin \theta}$$

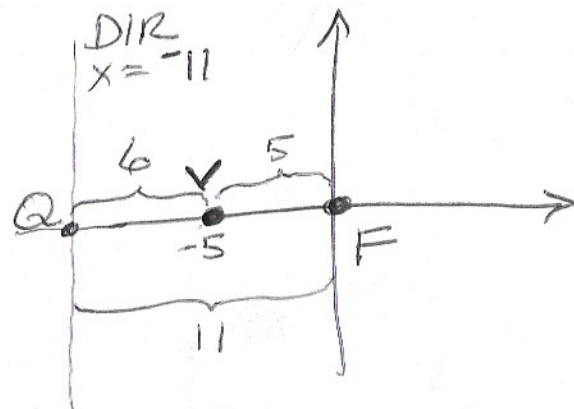
② VERTEX $(-5, 0)$ $p = 11$

DIRECTRIX IS 11 UNITS FROM POLE

$$r = \frac{ep}{1 - e \cos \theta} = \frac{\frac{5}{6} \cdot 11}{1 - \frac{5}{6} \cos \theta} = \frac{55}{6 - 5 \cos \theta}$$

$\uparrow$ $\uparrow$
 -11 $x =$

$$e = \frac{VF}{VQ} = \frac{5}{\frac{11}{6}} \rightarrow \text{ELLIPSE } 0 < e < 1$$



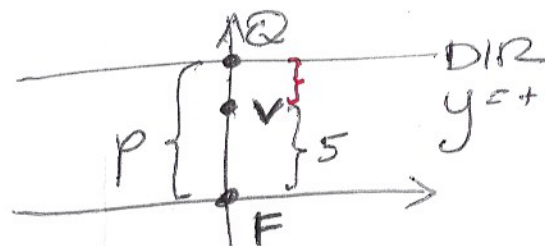
VERTEX BETWEEN
FOCUS + DIRECTRIX,
ON PERPENDICULAR
FROM FOCUS TO
DIRECTRIX

③ ECCENTRICITY = $\frac{3}{4} = e$

VERTEX = $(0, 5)$

$$r = \frac{ep}{1 + e \sin \theta} = \frac{\frac{3}{4} \cdot \frac{35}{3}}{1 + \frac{3}{4} \sin \theta} \cdot \frac{4}{4}$$

$$r = \frac{35}{4 + 3 \sin \theta}$$



$$e = \frac{VF}{VQ} = \frac{5}{p-5} = \frac{3}{4}$$

$$20 = 3(p-5)$$

$$20 = 3p - 15$$

$$35 = 3p$$

$$p = \frac{35}{3}$$

NOTE: INFORMATION GIVEN CAN INCLUDE

ECCENTRICITY

DIRECTRIX

VERTEX OR VERTICES

④ VERTICES $(0, 5), (0, 9)$

$$r = \frac{ep}{1 + e \sin \theta} = \frac{\frac{7}{2} \cdot \frac{45}{7}}{1 + \frac{7}{2} \sin \theta} = \frac{45}{2 + 7 \sin \theta}$$

$$e = \frac{V_1 F}{V_1 Q} = \frac{5}{p-5}$$

$$e = \frac{V_2 F}{V_2 Q} = \frac{9}{9-p}$$

$$\frac{5}{p-5} = \frac{9}{9-p}$$

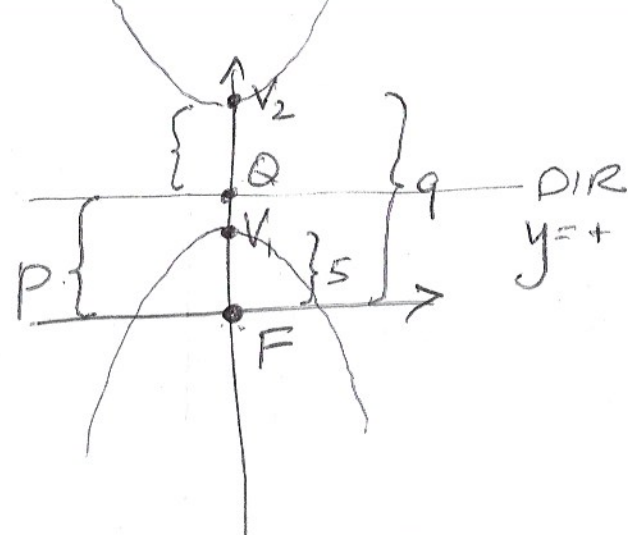
$$5(9-p) = 9(p-5)$$

$$45 - 5p = 9p - 45$$

$$90 = 14p$$

$$p = \frac{90}{14} = \frac{45}{7}$$

$$e = \frac{5}{\frac{45}{7} - 5} \cdot \frac{7}{7} = \frac{35}{45 - 35} = \frac{35}{10} = \frac{7}{2}$$

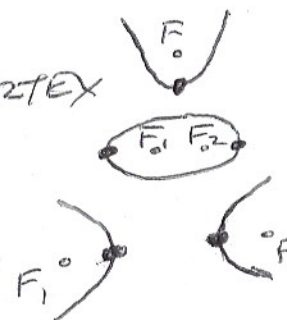


PARABOLA HAS 1 VERTEX

ELLIPSE

HYPERBOLA

2
2



ELLIPSE → DIR OUTSIDE

HYPERBOLA → DIR BETWEEN BRANCHES

DIR IS NEVER

AW AXIS OF SYM

↑
CAN BE PROVEN
BY ALGEBRA